Mission-driven Resource Allocation based on Subjective Input with Extra Level of Uncertainty

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***Abstract*—This paper presents a solution based on subjective logic to include 'second-order uncertainty' in objective functions for mission-driven resource allocation. When resources have to be optimized, one of the requirements is a probabilistic assessment of future scenarios. However, the end-users who have to convey such future assessment may not be completely confident about it. It can therefore be necessary to specify a confidence level together with this input. With subjective logic it is possible to describe such second-order uncertainty. We used it to describe the uncertainty of assessments (e.g. about future threats) and models (e.g. of sensor systems) within the optimization objective. As a result, the integrity of the optimization improves, because excluding important uncertainties from the optimality definition is not, actually, optimal. In the end, although it is complicated to incorporate different levels of uncertainty, the resulting analysis about mission outcomes is more genuine, it creates transparency towards the end-users, and eventually, results in a resource allocation solution that takes more aspects into account.**

I. Introduction

A key part of an automatic resource manager is the objective function [1], [2], [3]. An objective function should be able to quantify optimality (correctness) of a control solution. As a result, an objective function allows to compare many solutions with each other, and select the optimal one. It is obvious that the definition of such an objective function is crucial to actually manage resources optimally. However, there is only little discussion on the definition of objective functions (also defined as 'utility' or 'cost') in sensor management literature.

The main reason for the lack of discussion on the definition of the objective function is the complexity to prove its correctness. In fact, an objective function would be *the* method to judge if a solution is correct. If someone states "optimality is optimal, because my definition of optimality says so", then this is circular reasoning and not convincing. Nevertheless, we think that there is an approach to define a reasonably accurate objective function in a convincing and consistent manner. If this is possible, then a concrete method exist that can quantify how optimal system solutions are.

An extensive overview of objective functions is given in [1], [2], [3]. In short, the majority of current objective functions is based on system/sensing characteristics (e.g. [4], [5], [6], [7]). The advantage of such objective functions is that they can be easily linked with the controllable system parameters. However, the crucial problem with such objectives is that normally they are not linked with operational aspects. Fortunately, the number of studies that start to use objective functions that are more related to relevant operational aspects is increasing (e.g. [8], [9], [10], [11]).

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The objective functions that are currently the most successful in capturing optimality from a mission point of view are based on probabilistic risk [8], [10], expected-utility [11], [2] or prospect theory [3]. The approach consists of defining several assets (e.g. physical objects) that the end-user wants to obtain or retain. After the threats and opportunities have been analyzed probabilistically, the related risk, expected-utility or prospect can be minimized and/or maximized by allocating the system resources accordingly. This approach is much more consistent compared with satisfying (fixed) system/sensing characteristics. However, there are still several short-comings of such mission-driven objective functions.

The above mentioned mission-driven objective functions are based on probability measures. Probability is a mathematical language to describe and quantify uncertainty [12], which is required for the objective functions to output a real number (i.e. $\mathbb{R}$). However, not every type of uncertainty is captured in above functions. For example, let us assume that $P(S_z)$ is the probability that an asset z is secured. Based on the end-user's assessment and system models it is estimated that $P(S_z) = 0.8$. Such value is sufficient for a risk, expected-utility or prospect function to output the optimality of the control solution. However, how confident are we that this value 0.8 for $P(S_z)$ is correct? When pursuing this question we identify the 'uncertainty about probability' [13], which can also be called the 'second-order probability' about the original probability [14] or the 'epistemic uncertainty' [15]. It could be the case that the assessment of $P(S_z) = 0.8$ is actually poor and we cannot completely rely on it.

This paper investigates the use of subjective logic [14], [16] in order to include the second-order uncertainty in objective functions. This approach improves the integrity of the objective functions, because excluding important uncertainties from the optimality definition tends to unjustified confidence. For instance, $P(S_z)$ may depend on assessing future events. Future assessment is a complex process and actual quantification by end-users of $P(S_z)$ may result in a non-100% confidence that this value is correctly estimated. By using subjective logic we developed a solution that allows end-users and modelers to provide more complete 'expert opinion(s)' for enhancing the optimization process.

The content of this paper is as follows. In Section II a mission-driven objective function is defined based on the expected-utility approach. Section III discusses short-comings of probabilistic-based objective functions. The theory of Subjective Logic as a potential method to overcome these problems is discussed in Section IV. This theory is employed to assess second-order uncertainty for future assessment in Section V and model development in Section VI. Section VII combines all aspects in order for the objective functions to output a single real number that indicates optimality. This paper is concluded in Section VIII.

II. A Mission-driven Objective Function

An objective function is required to measure how much a system contributes to the mission. If $P(S_z)$ is the probability that a mission-critical asset z is secured when the mission ends, then the expected mission success is described by the expected-utility [17], [18]:

$$U_E = \sum_{z=1}^{Z} u_z P(S_z) \tag{1}$$

where Z is the number of assets and u_z is the utility of asset z. The utility is determined by the end-user, and the expected-utility describes the expected value of the assets at the end of the mission. For example, an assets can be threatened by a terrorist attack. As can be seen, the requirement of employing this mission-driven objective is that assets (nuclear plant, very-important-person, fugitive, etc.) can be defined. In order to secure the asset(s), it is required that operational tasks [2] are executed. Because operational tasks are linked with assets, the probability that an task is successfully executed is directly related to the mission success expectations:

$$P(S_z) = \prod_{k \in K_z} \begin{cases} (1 - P(E_k)\,(1 - P(S_{zk}|E_k))) & \text{if } k \in K_r \\ P(E_k)P(S_{zk}|E_k) & \text{if } k \in K_o \end{cases} \tag{2}$$

where the set K_z contains all operational tasks that are related to the asset z, set K_r contains all operational tasks for retaining an asset, and set K_o contains all operational tasks for (re-)obtaining an asset that was not in the end-user possession at the beginning of the mission. Probability $P(E_k)$ is the likelihood that the threat (if $k \in K_r$) or opportunity (if $k \in K_o$) specified by operational task k will occur and $P(S_{zk}|E_k)$ is the probability that the system is able to successfully cope with it for asset z.

Only probability $P(S_{zk}|E_k)$ can be directly influenced by the system. Thus, U_E can eventually be changed by adapting the system configuration. The probability that an operational task k is successfully executed for asset z is calculated by analyzing many possible scenarios:

$$P(S_{zk}|E_k) = \sum_{n=1}^{N_k} P(S_{zk}|X_{kn})P(X_{kn}|E_k) \tag{3}$$

where N_k is the number of test scenarios for operational task k, $P(S_{zk}|X_{kn})$ is the probability that the system successfully copes with scenario X_{kn}, $P(X_{kn}|E_k)$ is the probability this scenario will actually occur given that the related threat or opportunity E_k will occur specified by operational task k.

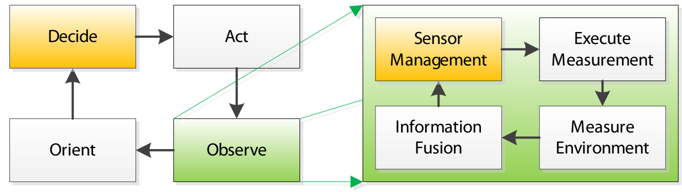

Fig. 1. System Architecture based on Observe Orient Decide Act (OODA).

We assume that the available systems are able to execute a set of operational tasks (e.g. air-defense, crowd-control) in the framework of the mission. The Observe, Orient, Decide and Act (OODA) loop [19] which is depicted in Fig. 1 can be used to identify which functions should be realized for a non-zero $P(S_{zk}|X_{kn})$. The original OODA loop identifies four functions for executing missions in general. However, these functions can again be decomposed. We illustrate this for the Observe function and re-use the OODA architecture to identify sub-functions: an 'information fusion', 'sensor management' and two other sub-functions. Thus, several (sub-)sub-systems are required to realize a complete successful system from an operational point of view.

The estimation of $P(S_{zk}|X_{kn})$ requires accurate models. Let us assume that the probability that the full system can cope with a possible scenario in perspective of asset z is given by:

$$P(S_{zk}|X_{kn}) = \prod_{m=1}^{M_k} P(S_{knm}|X_{kn}) \tag{4}$$

where $P(S_{knm}|X_{kn})$ is the probability that a system component m can deal with scenario X_{kn} related to operational task k, and M_k is the number of relevant components for operational task k. Thus, all relevant components should work in order to successfully cope with a scenario. The components can be defined on different levels. For example, on the level of 'sensors', 'effectors', 'environment', etc. Another level is also possible, for example: 'radar antenna', 'radar processing', etc.

III. Short-comings of Objective Functions

The objective function of the previous section has several strong advantages, such as that it is clearly related to operational impact. However, these two issues remain:

1) Probabilistic functions are only based on expectations that are mathematically defined. Thus, undefined possibilities are not included.
2) It is uncertain how accurate the quantification of expectations is, and this uncertainty is ignored by many probabilistic-based approaches.

This paper distinguishes between *expected*, *unexpected* and *undefined* events. If an event is undefined, then it is yet beyond our imagination [20], and there is no mention of a $P(E_k)$ at all. When it is defined, then it is expected or unexpected, namely $0 \leq P(E_k) \leq 1$. An event is expected instead of unexpected when $P(E_k)$ becomes higher than a certain threshold. For instance, unexpected may mean $P(E_k) < 0.1$. In any case, both expected and unexpected can be included in the analysis.

Let us consider an example for clarification. A threat is theoretically well defined, but completely unexpected, thus $P(E_k) = 0$. Then operational task k can be excluded from the analysis (or not). In any case (excluded or not), changing system performance $P(S_{zk}|E_k)$ does not affect the values for probability $P(S_z)$ and objective U_E. This is a good feature, because optimizing systems for completely unexpected events is superfluous from a mission point of view.

An objective function can only include aspects that can be defined mathematically. However, an event that is undefined can still happen [21]. To overcome this, it is sometimes suggested to "expect the unexpected". This triggers to brainstorm about unusual scenarios. However, note that it could also be understood as irrational, because "expecting" something that is still unexpected is likely to be useless. Further, it can also be argued that it is impossible to "expect the unexpected", because if someone considers (i.e. defines mathematically) the undefined it directly becomes (un)expected. Thus, these type of discussions demonstrate the need to properly define the employed terms.

Until now, solely the first issue is discussed, but the second issue is investigated in the rest of this paper. However, these two issues may overlap with each other. For instance, consider the uncertainty of including enough 'originally undefined' scenarios in the analysis. In any case, it is useful for a transparent analysis to discuss above issues separately as expressed by [12]:

> "Although uncertainty is uncertainty at a basic level, clearly some uncertainties are easier to assess than others. Therefore, it can be useful to think about certain types of distinctions among uncertainties."

For both issues there exist several solutions to partly solve them. For instance, possibility theory [21] is analyzed [22] as a solution to pay attention to undefined events, namely the first issue. We investigate the use of subjective logic to partly overcome the problem that is identified as the second issue.

IV. Introduction to Subjective Logic

This section discusses the theory of Subjective Logic [14] that is a generic belief reasoning calculus. Because it is a extension to binary logic and probability calculus, it is also compatible with them. The key advantage is that the theory has an extra uncertainty dimension: expert opinions are not solely based on a single likelihood, but have a 'second-order certainty' for the original likelihood.

In subjective logic, an (binomial) opinion about entity x is described as:

$$\omega_x = \{b_x; u_x; a_x\} \tag{5}$$

where b_x is the belief mass in support of x being true, u_x is the amount of uncommitted belief mass and uncertainty about x and a_x is the prior probability in the absence of commitment

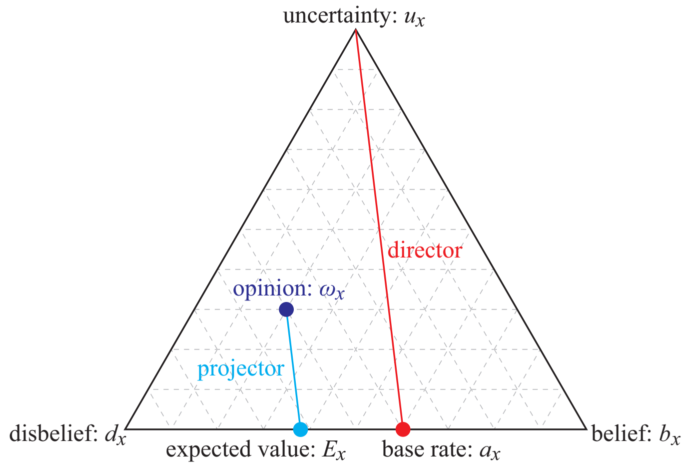

Fig. 2. Opinion $\omega_x = \{0.2; 0.3; 0.6\}$ plotted within Subjective Logic [14].

and base rate of x. It is defined that the sum of the belief, disbelief and uncertainty is equal to unity:

$$b_x + d_x + u_x = 1 \tag{6}$$

where d_x is the disbelief in x. In other words, if parameters b_x and u_x are determined, d_x is determined. This is the reason why we omitted disbelief d_x in the notation of ω_x, but it can be included [14]. The expected probability value is given by:

$$E(\omega_x) = b_x + a_x u_x \tag{7}$$

The base rate a_x refers to the category probability unconditioned on evidence, often referred to as prior probability. A simple example is the following. Assume that 1% of the public is a 'police officer' and 99% of the public is not, then the base rates in this case are 1% and 99%, respectively. When picking a random person, the prior probability of being a 'police officer' is 1%. However, additional information such as what type of clothes a person is wearing, at which location the person currently is, etc. can result in an opinion that the person is indeed a 'police officer' unequal to the 1% probability.

Fig. 2 depicts a triangle to visualize an opinion in Subjective Logic. Within the triangle there are three axes, namely, (i) belief, (ii) disbelief, and (iii) uncertainty. Any (binomial) opinion can be positioned within such a triangle. An example opinion ω_x about entity x is given with the purple point in Fig. 2. The location of the point visualizes the opinion in perspective of (dis)belief in x and the uncertainty about this (dis)belief. The closer the point is located to one of the three corners, the higher the related parameter is within the opinion. It is also possible to visualize the calculation of the expected value of the opinion. Namely, draw a line that is parallel to the director, which is based on the base rate, from the opinion location to the bottom.

Several types of opinions are possible. For instance, if $u_x \neq 0$, then it is a (partly) uncertain opinion, but if $u_x = 0$, then the opinion is dogmatic and it is equivalent to probabilities. On the other hand, if $u_x = 1$, then the opinion is vacuous.

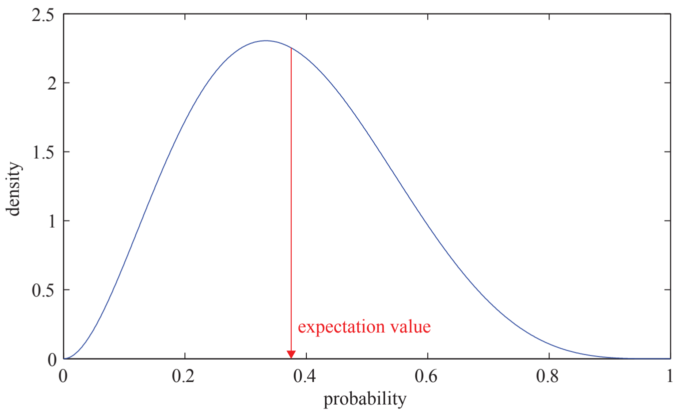

Fig. 3. Example opinion that is represented by a Beta function $f(x; 3, 5)$.

If $b_x = 1$, then the opinion is absolute, non-probabilistic and equivalent to binary 'true'.

It is also possible to map a binomial opinion ω_x to a Beta probability density function $f(x; \alpha, \beta)$ as described in [14]. In this case the parameters of $f(x; \alpha, \beta)$ are given as follows:

$$\alpha = \frac{Wb_x}{u_x} \tag{8}$$

$$\beta = \frac{Wd_x}{u_x} \tag{9}$$

where W is the prior weight and normally defined as $W = 2$ as explained in [14]. An example of the beta function $f(x; \alpha = 3, \beta = 5)$ for a $\omega_x = \{0.3; 0.2; a_x\}$ is given in Fig. 3.

To conclude, the subjective logic theory provides a framework to describe expert opinions and different degrees of uncertainty. Moreover, it is compatible with binary logic and probabilistic theory. Potentially the term 'uncertainty' can be replaced with the term 'confidence' as it is done in [15] to avoid potential confusion. Another advantage of the framework is that it provides consistent operators for opinions, which are used in the next sections.

V. Uncertainty during Future Assessment

The mission-driven objective function (1) of Section II is based on the expected-utility function. This means that the mission utility, that pertains to the future, is not directly optimized, which is impossible in a non-deterministic universe. However, the mission utility expectations are optimized, and by doing this, a better chance for mission success is obtained. Of course, this optimization relies on probabilities $P(E_k)$. Therefore, the correctness of probabilities $P(E_k)$ is crucial to actually maximize changes for mission success.

Future assessment determines which threats and opportunities become real in the future and quantify their likelihood. Thus, future assessment outputs $P(E_k)$. The actual assessment of future events is in practice complicated. Moreover, "there can be no experimental verification of its validity" [15] - or it can be done once, but then it is already too late. A remaining possibility is to use several inputs for future assessment, because this can enhance the confidence that the final assessment is correct [12]:

> "Since we generally believe, and empirical evidence supports, the notion that several heads are better than one, we often have information (e.g., probabilities, estimates) from a number of experts regarding the same events or variables."

Thus, let us assume that several experts provide their opinion about future events. The combination of several opinions into one statement is one of the possibilities in Subjective Logic [16] and the benefit is expressed similar as above:

> "In case of multiple belief sources it is assumed that each source represents partial evidence about the correct statement, so [...] fused belief from multiple sources provides stronger support for the correct statement [...] than [...] by beliefs from single sources alone."

Assume that person A is assessing the future and outputs $\omega^A(E_k)$ by positioning his opinion within the triangle as in Fig. 2. The resulting expert opinions is described as follows:

$$\omega^A(E_k) = \left\{b^A(E_k); u^A(E_k); a^A(E_k)\right\} \tag{10}$$

where $b^A(E_k)$ is the belief in E_k by person A, $u^A(E_k)$ is the confidence of person A and $a(E_k)$ is the assessed prior probability provided. Parameter $a(E_k)$ can, for example, be assumed equal over all the experts after a joint historical analysis (e.g. statistical occurrence of such threats or opportunities during previous similar missions).

There are two methods in subjective logic to fuse opinions [23]: cumulative fusion and averaging fusion. For future assessment, first the 'dependent opinions' are fused with averaging and the remaining 'independent opinions' are fused cumulatively. Opinions are dependent, for instance, when expert opinions are all based on reading social media messages. Independent opinions exist, for instance, when one expert's opinion is based on social media messages and another one is based on field reports (e.g. infiltrated spies).

A. *Averaging fusion*

The evidence, that opinions rely on, are assumed dependent during averaging fusion. Given two dependent opinions ω^A and ω^B, the opinion after averaging fusion is given by:

$$\omega^{A\underline{\diamond}B}(E_k) = \left\{b^{A\underline{\diamond}B}(E_k); u^{A\underline{\diamond}B}(E_k); a(E_k)\right\} \tag{11}$$

where symbol $\underline{\diamond}$ expresses the fusion of two observers into a single imaginary observer. Thus, if the observers are A and B, then the imaginary observer is denoted as $A\underline{\diamond}B$. When at least one of the persons is not 100% certain, then the resulting opinion is given by [23]:

$$b^{A\underline{\diamond}B} = \frac{b^A u^B + b^B u^A}{u^A + u^B} \tag{12}$$

$$u^{A\underline{\diamond}B} = \frac{2u^A u^B}{u^A + u^B} \tag{13}$$

when each person is 100% certain and the opinions are not identical, then there is a conflict and the result is calculated differently [23]. The averaging fusion rule is commutative and idempotent, but not associative. Idempotent means that a belief argument fused with itself should always produce the same belief argument.

The non-associative characteristic of this fusion operator can be problematic in our type of problems. For instance, if there are three opinions, which sequence should be used to fuse the opinions? However, it is also debatable that averaging fusion is needed in practice, because when several experts are generating an opinion based on the same evidence, then probably they can also formulate together a single opinion as a group.

B. *Cumulative fusion*

For cumulative fusion it is assumed that the evidence, that opinions rely on, is independent. Let us assume two independent opinions ω^A and ω^B, then the cumulative fused opinion is described as:

$$\omega^{A\diamond B}(E_k) = \left\{b^{A\diamond B}(E_k); u^{A\diamond B}(E_k); a(E_k)\right\} \tag{14}$$

where symbol $\diamond$ denotes the fusion of two observers into a single imaginary observer. When the two observers are A and B, then the imaginary observer is expressed as $A \diamond B$. If at least one of the persons is not 100% certain about his/her assessment, then the resulting opinion is given by [23]:

$$b^{A\diamond B} = \frac{b^A u^B + b^B u^A}{u^A + u^B - u^A u^B} \tag{15}$$

$$u^{A\diamond B} = \frac{u^A u^B}{u^A + u^B - u^A u^B} \tag{16}$$

when each person is 100% certain and the opinions are not identical, then there is a conflict and another set of equations is given in [23]. The cumulative fusion operator is commutative, associative and non-idempotent. The non-idempotent means that a partially uncertain belief argument fused with itself should produce a fusion result with less uncertainty. Therefore, opinions that rely on the same evidence should not be fused cumulatively.

VI. Uncertainty during Model Development

When a threat or opportunity is actually occurring during the mission, the system should be able to deal with it. The successful handling can depend on many system components as discussed in Section II. For instance, the detection of objects may depend on radar systems. When considering a radar, the exact estimation of the (cumulative) detection probability depends on many aspects (e.g. antenna characteristics, propagation loss in the environment) that have to be incorporated in the model. In the end, the model is a representation of the reality and there is practically always a mismatch. Thus, a certain level of confidence exist at the modeler that the calculated system performance is accurate, as stated in [15]:

> "Since any model is an approximate representation, it follows that there must be some (epistemic) uncertainty associated with the formulation, and predictions, of the model."

This model uncertainty is again a second order uncertainty which should be considered during system optimization. As discussed in [12]:

> "When we are uncertain about a model, then assessments of probability distributions for the model parameters can be made conditional upon the model being appropriate, and model uncertainty can be treated separately."

A model has to be developed for each system component m and for each model m three components have to be provided. Firstly, belief $b(S_{knm}|X_{kn})$ which should be linked to the model output (e.g. the probability that a threatening object will be detected-in-time by a sensor). Secondly, the modeler, who developed the model, should quantify its confidence (or trust) in the model: uncertainty $u(S_{knm}|X_{kn})$. It is possible that the modeler only has to provide one value U_m for all scenarios, thus $\forall m : u(S_{knm}|X_{kn}) = U_m$. Thirdly, the base rate $a(S_{knm}|X_{kn})$ which is challenging to estimate. A possibility is an extensive measurement campaign that includes real-life self-constructed scenarios for testing. After such measurement campaign it can be expected that the model is improved by the modeler and simultaneously that the modeler's confidence level is changing. In any case, the opinion that system m is able to cope with scenario n of operational task k is given by:

$$\omega(S_{knm}|X_{kn}) = \{b(S_{knm}|X_{kn}); u(S_{knm}|X_{kn}); a(S_{knm}|X_{kn})\} \tag{17}$$

All the modeler opinions $\omega(S_{knm}|X_{kn})$ have to be combined into a single $\omega(S_{zk}|X_{kn})$. In this paper, the final performance is given by the multiplication of the individual system component performances as described by (4). In this case, the required operator is the (binomial) multiplication [24].

A. *Normal Binomial Multiplication*

In subjective logic, the symbols $\wedge$ and $\cdot$ are used to denote the multiplication of opinions: $\omega_{x\wedge y} = \omega_x \cdot \omega_y$. The parameters of $\omega_{x\wedge y}$ are given by:

$$b_{x\wedge y} = b_x b_y + \frac{(1-a_x)a_y b_x u_y + a_x(1-a_y)u_x b_y}{1 - a_x a_y} \tag{18}$$

$$u_{x\wedge y} = u_x u_y + \frac{(1-a_y)b_x u_y + (1-a_x)u_x b_y}{1 - a_x a_y} \tag{19}$$

$$a_{x\wedge y} = a_x a_y \tag{20}$$

The multiplication operators are commutative and associative. When the opinions contain degrees of uncertainty, the multiplication operators will produce opinions that have a correct expectation value but possibly with approximate variance for Beta probability distributions [14].

VII. Integrating Extra Level of Uncertainty

In Sections V and VI, $\omega(E_k)$ and $\omega(S_{zk}|X_{kn})$ have been derived. The remaining task is to integrate them in the objective function, that should still output a single $\mathbb{R}$ value in order to unambiguously select the optimal system configuration.

The opinion that task k is successfully executed given an attack is defined based on (3) as follows:

$$\omega(S_{zk}|E_k) = \sum_{x=1}^{X_k} \omega(S_{zk}|X_{kn}) P(X_{kn}|E_k) \tag{21}$$

In the above case, $P(X_{kn}|E_k)$ is not replaced with a $\omega(X_{kn}|E_k)$, which may include some second-order uncertainty. Of course, both solutions are possible, because Subjective Logic is compatible with the original probabilistic logic.

The addition of two opinions $\omega_{x\cup y} = \omega_x + \omega_y$ is defined as follows [14]:

$$b_{x\cup y} = b_x + b_y \tag{22}$$

$$u_{x\cup y} = \frac{a_x u_x + a_y u_y}{a_x + a_y} \tag{23}$$

$$a_{x\cup y} = a_x + a_y \tag{24}$$

Based on (2) the opinion that an asset z is secured is defined as follows:

$$\omega(S_z) = \prod_{k\in K_z} \left\{ \begin{array}{ll} \neg\left(\omega(E_k)(\neg\omega(S_{zk}|E_k))\right) & \text{if } k \in K_r \\ \omega(E_k)\omega(S_{zk}|E_k)) & \text{if } k \in K_o \end{array} \right. \tag{25}$$

where $\neg$ is the subjective logic complement notation and simply defined as:

$$b_{\neg x} = d_x \tag{26}$$

$$u_{\neg x} = u_x \tag{27}$$

$$a_{\neg x} = 1 - a_x \tag{28}$$

As a result, an opinion $\omega(S_z) = \{b(S_z); u(S_z); a(S_z)\}$ exists for each asset. Finally, it is proposed to maximize the following 'subjective-expected-utility':

$$s(U_E) = \sum_{z=1}^{Z} u_z E\left(\omega(S_z)\right) \tag{29}$$

$$s(U_E) = \sum_{z=1}^{Z} u_z b(S_z) + \sum_{z=1}^{Z} u_z a(S_z) u(S_z) \tag{30}$$

where function $E(\omega_x)$ is the expected probability value of opinion ω_x as defined in (7).

The resulting objective function includes the second-order uncertainty. It has transformed the final opinions to expected values in order to compute a single value that can express the optimality of system solutions. An advantage of the presented approach is that also the total uncertainty about the mission outcome estimation can be provided back to the end-user:

$$u(U_E) = \sum_{z=1}^{Z} u_z u(S_z) \tag{31}$$

Value $u(U_E)$ can be used to indicate how uncertain the optimization process is about $s(U_E)$. If there exist no model uncertainties or the binomial multiplication, which uses b_x and b_y to determine $u_{x\wedge y}$, is not required for combining the model uncertainties, then $u(U_E)$ is independent on the selected system configuration. In this case, the uncertainty of the mission outcome $u(U_E)$ only depends on the uncertainty of the future assessment, and thus, $u(U_E)$ cannot be changed by the system.

Potentially, the subjective logic based approach can be combined with a prospect theory based resource allocation approach [3]. In this case, the objective function is given by:

$$s(U_P) = \sum_{z=1}^{Z} v(u_z) w\left(E(S_z)\right) \tag{32}$$

where function $v(.)$ translates the utilities into 'subjective' values and function $w(.)$ translates probabilities into weights. In this way the 'subjective-prospect' objective can be maximized. The uncertainty in the prospect is calculated in a similar way:

$$u(U_P) = \sum_{z=1}^{Z} v(u_z) u(S_z) \tag{33}$$

This concludes the incorporation of the second-order uncertainty of future assessment and model development in the optimization objective for mission-driven resource allocation.

VIII. Conclusion

The mission-driven objective function are enhanced by incorporating the uncertainty of future events and model outputs. The key obtained advantage is that resource allocation can now take into account these types of uncertainties during its optimization, and moreover, output its uncertainty of its optimality of the provided configuration. We expect that end-users will appreciate the extra uncertainty dimension for conveying their future assessment, and allowing modelers to express their confidence. On the other hand, we should be aware that there is a possibility that end-users become confused, because probability theory is used to parametrize and quantify both types of uncertainty, but as expressed in [15]:

> "it is important to distinguish between the two, not only because it can impact the answer being given to a decision maker, and hence have an impact on the decision outcomes"

It is clear that in the above quote the term 'decision maker' can be replaced with 'automatic resource allocator'. To conclude, although it is more complicated to incorporate different levels of uncertainty, the resulting analysis about mission outcomes is more genuine, it creates transparency towards the end-users, and eventually, results in a system configuration solution that takes more aspects into account.

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REFERENCES

[1] T. de Groot, O. Krasnov, and A. Yarovoy, "Generic utility definition for mission-driven resource allocation," in *IET International Conference on Radar Systems*, 2012, pp. 1–5.

[2] ——, "Mission-driven management and allocating operational tasks to reconfigurable sensors," *submitted to IEEE Transactions on Aerospace and Electronic Systems*, vol. x, no. x, p. x, 2014.

[3] ——, "Mission-driven sensor management based on expected-utility and prospect objectives," in *International Conference on Information Fusion*, 2014, pp. 1–8.

[4] C. Baker and A. Hume, "Netted radar sensing," *IEEE Aerospace and Electronic Systems Magazine*, vol. 18, no. 2, pp. 3–6, 2003.

[5] J. Wintenby and V. Krishnamurthy, "Hierarchical resource management in adaptive airborne surveillance radars," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 42, no. 2, pp. 401–420, 2006.

[6] C. Yang, L. Kaplan, and E. Blasch, "Performance measures of covariance and information matrices in resource management for target state estimation," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 3, pp. 2594–2613, 2012.

[7] M. Zink, E. Lyons, D. Westbrook, and J. Kurose, "Closed-loop architecture for distributed collaborative adaptive sensing of the atmosphere: meteorological command and control," *International Journal of Sensor Networks*, vol. 7, no. 1, pp. 4–18, 2010.

[8] F. Johansson and G. Falkman, "Real-time allocation of defensive resources to rockets, artillery, and mortars," in *International Conference on Information Fusion*, 2010, pp. 1–8.

[9] D. Papageorgiou and M. Raykin, "A risk-based approach to sensor resource management," in *Advances in Cooperative Control and Optimization*. Springer, 2007, pp. 129–144.

[10] F. Bolderheij, "Mission-driven sensor management - analysis, design, implementation and simulation," Ph.D. dissertation, Delft University of Technology, 2007.

[11] M. Ditzel, L. Kester, S. van den Broek, and M. van Rijn, "Cross-layer utility-based system optimization," in *International Conference on Information Fusion*, 2013, pp. 507–514.

[12] R. L. Winkler, "Uncertainty in probabilistic risk assessment," *Reliability Engineering & System Safety*, vol. 54, no. 2, pp. 127–132, 1996.

[13] T. Bedford and R. Cooke, *Probabilistic risk analysis: foundations and methods*. Cambridge University Press, 2001.

[14] A. Jøsang, *Subjective Logic (Draft)*. University of Oslo, 2013, available at http://folk.uio.no/josang/ [last accessed 04-02-2014].

[15] G. W. Parry, "The characterization of uncertainty in probabilistic risk assessments of complex systems," *Reliability Engineering & System Safety*, vol. 54, no. 2, pp. 119–126, 1996.

[16] A. Jøsang, P. C. Costa, and E. Blash, "Determining model correctness for situations of belief fusion," in *International Conference on Information Fusion*, 2013, pp. 1–8.

[17] M. Friedman and L. J. Savage, "The utility analysis of choices involving risk," *Journal of Political Economy*, vol. 56, no. 4, pp. 279–304, 1948.

[18] P. J. Schoemaker, "The expected utility model: Its variants, purposes, evidence and limitations," *Journal of Economic Literature*, pp. 529–563, 1982.

[19] J. R. Boyd, "The essence of winning and losing," *Unpublished lecture notes*, 1996.

[20] M. Ramana, "Beyond our imagination: Fukushima and the problem of assessing risk," *Bulletin of Atomic Scientists, 19th April*, 2011.

[21] L. A. Zadeh, "Fuzzy sets as a basis for a theory of possibility," *Fuzzy sets and systems*, vol. 100, pp. 9–34, 1999.

[22] A. A. Alola, M. Tunay, and V. Alola, "Analysis of possibility theory for reasoning under uncertainty." *International Journal of Statistics & Probability*, vol. 2, no. 2, 2013.

[23] A. Jøsang, J. Diaz, and M. Rifqi, "Cumulative and averaging fusion of beliefs," *Information Fusion*, vol. 11, no. 2, pp. 192–200, 2010.

[24] A. Jøsang and D. McAnally, "Multiplication and comultiplication of beliefs," *International Journal of Approximate Reasoning*, vol. 38, no. 1, pp. 19–51, 2005.